INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only marks for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators and dictionaries must NOT be used. There is no formulae booklet for this test.

Please wait to be told you may begin before turning this page.

This question paper consists of 21 printed pages and 3 blank pages.
1 Given that
\[
\frac{dy}{dx} = 3x^2 - \frac{2-3x}{x^3}, \quad x \neq 0
\]
and \(y = 5\) when \(x = 1\), find \(y\) in terms of \(x\).

A \(y = \frac{1}{3}x^3 + x^{-2} - 3x^{-1} + 6 \frac{2}{3}\)
B \(y = x^3 + \frac{1}{2}x^{-2} - 3x^{-1} + 6 \frac{1}{2}\)
C \(y = x^3 + x^{-2} - 3x^{-1} + 6\)
D \(y = x^3 + x^{-2} - x^{-1} + 4\)
E \(y = x^3 + 2x^{-2} - x^{-1} + 3\)
F \(y = 3x^3 + x^{-2} - x^{-1} + 2\)

\[
\int 3x^2 - 2x^{-3} + 3x^{-2} \, dx = \frac{3x^3}{3} + 2x^{-2} - 3x^{-1} + c
\]
\[
= x^3 + x^{-2} - 3x^{-1} + c
\]

When \(x = 1\), \(y = 5\) so:
\[
y = x^3 + x^{-2} - 3x^{-1} + c
\]
\[
5 = 1^3 + 1^{-2} - 3 \cdot 1^{-1} + c
\]
\[
5 = 1 + 1 - 3 + c
\]
\[
c = 6
\]
\[
\therefore y = x^3 + x^{-2} - 3x^{-1} + 6
\]
The function $f$ is given by

$$f(x) = \left(\frac{2}{x} - \frac{1}{2x^2}\right)^2 \quad (x \neq 0)$$

What is the value of $f''(1)$?

A  $-3$
B  $-1$
C  5
D  17
E  29
F  80

C

$$f''(x) = \left(\frac{2}{x} - \frac{1}{2x^2}\right)^2$$

$$= \frac{4}{x^2} - \frac{2}{2x^3} - \frac{2}{2x^3} + \frac{1}{4x^4}$$

$$= \frac{4}{x^2} - \frac{4}{2x^3} + \frac{1}{4x^4}$$

$$= \frac{4}{x^2} - \frac{2}{x^3} + \frac{1}{4x^4}$$

$$= 4x^{-2} - 2x^{-3} + \frac{1}{4}x^{-4}$$

So $f''(x) = -8x^{-3} + 6x^{-4} - x^{-5}$

$$f''(x) = 24x^{-4} - 24x^{-5} + 5x^{-6}$$

Then $f''(1) = 24 - 24 + 5 = 5$
Call the second line $l'$

3 A line $l$ has equation $y = 6 - 2x$

A second line is perpendicular to $l$ and passes through the point $(-6, 0)$.

Find the area of the region enclosed by the two lines and the $x$-axis.

\[ l' \text{ is } y = 6 - 2x \quad \text{Gradient is } -2 \]

\[ \therefore \text{gradient of } l' \text{ is } \frac{1}{2} \]

For $l'$ we know $y = mx + c$

\[ = \frac{1}{2}x + c \]

Since $(-6, 0)$ lies on $l'$, we get

\[ 0 = \frac{1}{2}(-6) + c \]

\[ c = 3 \]

Hence the equation of $l'$ is $y = \frac{1}{2}x + 3$

The lines $y = 6 - 2x$ and $y = \frac{1}{2}x + 3$ intersect so

\[ 6 - 2x = \frac{1}{2}x + c \]

\[ y = \frac{1}{2}x + 3 \]

\[ 2.5x = 3 \]

\[ x = \frac{6}{5} \]

So

\[ y = 6 - \frac{6}{5} \times 2 = \frac{18}{5} \]

\[ \therefore \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (3 + 6) \times \frac{18}{5} = 16 \frac{1}{2} \]
When \((3x^2 + 8x - 3)\) is multiplied by \((px - 1)\) and the resulting product is divided by \((x + 1)\), the remainder is 24.

What is the value of \(p\)?

\[3x^2 + 8x - 3 = (3x-1)(x + 3)\]

A  \(-4\)

B  \(2\)

If we let \(x = -1\) then:

\[(3x-1)(x + 3)(px - 1) = (-3-1)(-1+3)(-p+1)\]

\[= -4 \times 2 \times (-p + 1)\]

\[= -8(-p + 1)\]

\[= 8p - 8\]

Remainder is 24 when divided by \((x+1)\) so

\[8p - 8 = 24\]

\[8p = 16\]

\[p = 2\]
5. \( S \) is the complete set of values of \( x \) which satisfy both the inequalities
\[ x^2 - 8x + 12 < 0 \text{ and } 2x + 1 > 9 \]

The set \( S \) can also be represented as a single inequality.

Which one of the following single inequalities represents the set \( S \) ?

A. \((x^2 - 8x + 12)(2x + 1) < 0\)
B. \((x^2 - 8x + 12)(2x + 1) > 0\)
C. \(x^2 - 10x + 24 < 0\)
D. \(x^2 - 10x + 24 > 0\)
E. \(x^2 - 6x + 8 < 0\)
F. \(x^2 - 6x + 8 > 0\)
G. \(x < 2\)
H. \(x > 6\)

For (1):
\[ x^2 - 8x + 12 = (x - 6)(x - 2) \]

If \( y = (x - 6)(x - 2) \) then it looks like \[ \text{graph}\]

So \((x - 6)(x - 2) < 0\) when \(2 < x < 6\)

For (2):
\[ 2x + 1 > 9 \]
\[ 2x > 8 \]
\[ x > 4 \]

When both \( x > 4 \) and \( 2 < x < 6 \) are true, we have \( 4 < x < 6 \)

This is given by \((x - 4)(x - 6) < 0\) i.e. \(x^2 - 10x + 24 < 0\)
6 A tangent to the circle $x^2 + y^2 = 144$ passes through the point $(20, 0)$ and crosses the positive $y$-axis.

What is the value of $y$ at the point where the tangent meets the $y$-axis?

**A** 12

**B** 15

**C** $\frac{49}{3}$

**D** 20

**E** $\frac{64}{3}$

**F** $\frac{80}{3}$

Equation of $l$ is of the form $y = mx + c$

It passes through $(20, 0)$ so

$0 = 20m + c$

$c = -20m$

i.e. $y = mx - 20m$

$= m(x - 20)$

If we sub this into $x^2 + y^2 = 144$ we get:

$x^2 + (m(x - 20))^2 = 144$

$x^2 + m^2(x^2 - 40x + 400) = 144$

$(m^2 + 1)x^2 - 40m^2x + (400m^2 - 144) = 0$

For this to have one root we need $b^2 - 4ac = 0$, i.e.

$(-40m^2)^2 - 4(m^2 + 1)(400m^2 - 144) = 0$

$1600m^4 - 1600m^4 - 1024m^2 + 576 = 0$

$-1024m^2 = -576$

$m^2 = \frac{9}{16}$

$m = \pm \frac{3}{4}$

but as we know the gradient is negative, i.e. $m < 0$, then $m = -\frac{3}{4}$

So we get $y = -\frac{3}{4}(x - 20)$

Tangent meets the $y$-axis at $x = 0$, i.e. $y = -\frac{3}{4}x - 20 = 15$
The first three terms of an arithmetic progression are \( p, q \) and \( p^2 \) respectively, where \( p < 0 \).

The first three terms of a geometric progression are \( p, p^2 \) and \( q \) respectively.

Find the sum of the first 10 terms of the arithmetic progression.

\[
\begin{align*}
\text{A} & \quad \frac{23}{8} \\
\text{B} & \quad \frac{95}{8} \\
\text{C} & \quad \frac{115}{8} \\
\text{D} & \quad \frac{185}{8}
\end{align*}
\]

In the GP, from (2) we get \( p = r \)

\[
\begin{align*}
\text{AP} & \quad (3) \quad q = p^3 \\
\end{align*}
\]

Using \( q = p^3 \) in the AP we get:

\[
\begin{align*}
q &= p + d \\
p^3 &= p + d \\
d &= p^3 - p \\
2d &= 2p^3 - 2p
\end{align*}
\]

then we have:

\[
\begin{align*}
2p^3 - 2p &= p^2 - p \\
2p^3 - p^2 - p &= 0 \\
p(2p^2 - p - 1) &= 0 \\
p(2p + 1)(p - 1) &= 0
\end{align*}
\]

so \( p = 0, -\frac{1}{2} \) and 1

we were told \( p < 0 \) so \( p = -\frac{1}{2} \)

\[
\begin{align*}
d &= p^3 - p = -\frac{1}{8} + \frac{1}{2} = \frac{3}{8} \\
a &= p = -\frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
S_n &= \frac{n(2a + (n-1)d)}{2} \\
S_{10} &= \frac{10(2(-\frac{1}{2}) + 9 \times \frac{3}{8})}{2} = \frac{95}{8}
\end{align*}
\]
8 Find the complete set of values of \( x \), with \( 0 \leq x \leq \pi \), for which

\[
(1 - 2 \sin x) \cos x \geq 0
\]

\[\text{When } \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}\]

\[\text{when } x = 0, 1 - 2 \sin x = 1\]

\[\text{Hence } 1 - 2 \sin x \neq 0 \text{ when}\]

\[0 \leq x \leq \frac{\pi}{2} \text{ and } \frac{\pi}{6} \leq x \leq \pi\]

\[\text{when } x = \pi, 1 - 2 \sin x = -1\]

\[\text{Hence } 1 - 2 \sin x \leq 0 \text{ when}\]

\[\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}\]

\[\cos x = 0 \text{ when } x = \frac{\pi}{2}\]

\[\cos x \neq 0 \text{ when } 0 \leq x \leq \frac{\pi}{2}\]

\[\cos x \leq 0 \text{ when } \frac{\pi}{2} \leq x \leq \pi\]

Combining all of these expressions we get

\[(1 - 2 \sin x) \cos x \geq 0 \text{ when } 0 \leq x \leq \frac{\pi}{6} \leq 0 \]

\[\leq 0 \quad \frac{\pi}{2} \leq x \leq \frac{5\pi}{6}\]
A circle has equation $x^2 + y^2 - 18x - 22y + 178 = 0$

A regular hexagon is drawn inside this circle so that the vertices of the hexagon touch the circle.

What is the area of the hexagon?

Circle $x^2 + y^2 - 18x - 22y + 178 = 0$

$(x^2 - 18x) + (y^2 - 22y) = -178$

$(x-9)^2 - 81 + (y-11)^2 - 121 = -178$

$(x-9)^2 + (y-11)^2 = 24$

Centre $(9,11)$, radius $\sqrt{24} = 2\sqrt{6}$

A hexagon is 6 triangles

Triangle height $h^2 = (2\sqrt{6})^2 - (\sqrt{6})^2$

$= 4 \times 6 - 6$

$= 3 \times 6$

$= 18$

$h = \sqrt{18}$

$= 3\sqrt{2}$

Triangle area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2\sqrt{6} \times 3\sqrt{2} = 3\sqrt{2} \times \sqrt{6} = 6\sqrt{3}$

Hexagon area = $6 \times 6\sqrt{3} = 36\sqrt{3}$
A curve $C$ has equation $y = f(x)$ where

$$f(x) = p^3 - 6p^2x + 3px^2 - x^3$$

and $p$ is real.

$$= -x^3 + 3p \cdot x^2 - 6p^2 \cdot x + p^3$$

The gradient of the normal to the curve $C$ at the point where $x = -1$ is $M$.

What is the greatest possible value of $M$ as $p$ varies?

A $\frac{3}{2}$

$$(x) = -3x^2 + 6p \cdot x - 6p^2$$

B $\frac{2}{3}$

$$f'(-1) = -3 - 6p - 6p^2 \quad \text{gradient of } C$$

C $\frac{1}{2}$

Gradient of normal = $\frac{-1}{-6p^2 - 6p - 3} = \frac{1}{3(2p^2 + 2p + 1)}$

D $\frac{1}{4}$

$$\frac{d}{dp}(2p^2 + 2p + 1) = 4p + 2$$

E $\frac{2}{3}$

$$4p + 2 = 0$$

$$(2p^2 + 2p + 1)$$

F $\frac{3}{2}$

$$(p = -\frac{1}{2})$$

so the denominator is minimised (i.e. the gradient maximised)

when $p = -\frac{1}{2}$ and denominator is $\frac{1}{2}$

so greatest gradient of normal $= \frac{1}{3 \times \frac{1}{2}} = 2\frac{2}{3}$
11 The sequence $x_n$ is defined by the rules

$$x_1 = 7$$

$$x_2 = 3$$

$$x_3 = 1$$

$$x_{n+1} = \frac{23x_n - 53}{5x_n + 1}$$

The first three terms in the sequence are 7, 3, 1

What is the value of $x_{100}$?

A) 7
B) 3
C) 1
D) 3
E) 7

cycle length of 4 (7, 3, 1, -5)

$$\frac{100}{4} = 25$$ hence $x_{100} = -5$
The polynomial function $f(x)$ is such that $f(x) > 0$ for all values of $x$.

Given $\int_{2}^{4} f(x) \, dx = A$, which one of the following statements must be correct?

A $\int_{0}^{2} [f(x + 2) + 1] \, dx = A + 1$

B $\int_{0}^{2} [f(x + 2) + 1] \, dx = A + 2$

C $\int_{2}^{4} [f(x + 2) + 1] \, dx = A + 1$

D $\int_{2}^{4} [f(x + 2) + 1] \, dx = A + 2$

E $\int_{4}^{6} [f(x + 2) + 1] \, dx = A + 1$

F $\int_{4}^{6} [f(x + 2) + 1] \, dx = A + 2$

$\int_{0}^{2} (f(x+2)+1) \, dx = \int_{0}^{2} f(x+2) \, dx$

$= \int_{0}^{2} 1 \, dx$

$= A + 2$
13 In the expansion of \((a + bx)^5\) the coefficient of \(x^4\) is 8 times the coefficient of \(x^2\).

Given that \(a\) and \(b\) are non-zero positive integers, what is the smallest possible value of \(a + b\)?

\[
(a + bx)^5 = a^5 + \binom{5}{1} a^4 (bx) + \binom{5}{2} a^3 (bx)^2 + \binom{5}{3} a^2 (bx)^3 + \binom{5}{4} a (bx)^4 + \binom{5}{5} (bx)^5
\]

\[= a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + \ldots\]

Co-efficient of \(x^4 = 5ab^4\)

\[x^2 = 10a^3b^2\]

We’re told \(8(10a^3b^2) = 5ab^4\) \[\Rightarrow 80a^3b^2 = 5ab^4\]

\[80a^2 = 5b^2\]

\[16a^2 = b^2\]

\[4a = b\]

\[a+b = a+4a = 5a\]

If both nonzero positive then the smallest \(a\) or \(b\) can be is 1, ie. \(\max(a+b = 5a) = 5\)
The solution of the simultaneous equations

\begin{align*}
(1) & \quad 2^x + 3 \times 2^y = 3 \\
(2) & \quad 2^{2x} - 9 \times 2^{2y} = 6
\end{align*}

is \( x = p, \ y = q \).

Find the value of \( p - q \)

Let \( 2^x = a \) then \( 2^y = b \)

A \[ \frac{5}{12} \]

B \[ \frac{7}{3} \]

\[(1) \quad a + 3b = 3 \quad \Rightarrow \quad a = 3 - 3b \quad \text{& sub into (2)} \]

\[(2) \quad a^2 - 9b^2 = 6 \]

C \[ \log_2 \frac{5}{12} \]

D \[ \log_2 \frac{7}{3} \]

\[(3 - 3b)^2 - 9b^2 = 6 \]

\[(3 - 3b)(3 - 3b) - 9b^2 = 6 \]

E \[ \log_2 9 \]

\[ q - 18b + 9b^2 - 9b^2 = 6 \]

F \[ \log_2 15 \]

\[ q - 18b = 6 \]

\[ 3 = 18b \]

\[ b = \frac{3}{18} = \frac{1}{6} \]

\[ \text{so} \quad a = 3 - 3 \left( \frac{1}{6} \right) = 3 - \frac{1}{2} = 2 \frac{1}{2} = \frac{5}{2} \]

So \( a = \frac{5}{2} = 2^x \) then \( x = \log_2 a = \log_2 \frac{5}{2} = p \)

\[ b = \frac{1}{6} = 2^y \]

\[ y = \log_2 b = \log_2 \frac{1}{6} = q \]

So \( p - q = \log_2 \frac{5}{2} - \log_2 \frac{1}{6} = \log_2 \left( \frac{5}{2} \div \frac{1}{6} \right) = \log_2 15 \)
It is given that \( f(x) = -2x^2 + 10 \).

Consider the following three curves:

(1) \( y = f(x) \) underestimate

(2) \( y = f(x + 1) \) underestimate

(3) the curve \( y = f(x + 1) \) reflected in the line \( y = 6 \) overestimate

The trapezium rule is used to estimate the area under each of these three curves between \( x = 0 \) and \( x = 1 \).

State whether the trapezium rule gives an overestimate or underestimate for each of these areas.

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\[ f(x) = 3x^2 + 12x + 4 \]
\[ g(x) = x^3 + 6x^2 + 9x - 8 \]
\[ f'(x) = 6x + 12 \]
\[ g'(x) = 3x^2 + 12x + 9 \]
\[ = 3(x^2 + 4x + 3) \]
\[ = 3(x+1)(x+3) \]

A function \( f(x) \) is increasing when \( f'(x) > 0 \)

decreasing \[ f'(x) \leq 0 \]

16 The functions \( f \) and \( g \) are given by \( f(x) = 3x^2 + 12x + 4 \) and \( g(x) = x^3 + 6x^2 + 9x - 8 \).

What is the complete set of values of \( x \) for which one of the functions is increasing and the other decreasing?

1) If \( f(x) \) is increasing \& then we have:
\[ g(x) \] decreasing

A \[ x \geq -1 \]

B \[ x \leq -1 \]

C \[ -3 \leq x \leq -2, \ x \geq -1 \]

D \[ x \leq -2, \ x \geq -1 \]

E \[ x \leq -3, \ -2 \leq x \leq -1 \]

Combine those \& get \[ -2 \leq x \leq -1 \]

F \[ x \leq -3, \ x \geq -2 \]

G \[ -2 \leq x \leq -1 \]

2) If \( f(x) \) is decreasing \& then we have:
\[ g(x) \] is increasing

\[ 6x + 12 \leq 0 \]
\[ 6x \leq -12 \]
\[ x \leq -2 \]

\[ 3(x+1)(x+3) \geq 0 \]

\[ x \leq -3 \] or \[ x \geq -1 \]

Combine those \& get \[ x \leq -3 \]

So the complete set is \[ x \leq -3 \] or \[ -2 \leq x \leq -1 \]
The two functions \( F(n) \) and \( G(n) \) are defined as follows for positive integers \( n \):

\[
F(n) = \frac{1}{n} \int_0^n (n-x) \, dx = \frac{1}{n} \left( nx - \frac{1}{2} x^2 \right) \Big|_0^n = \frac{1}{n} \left( n^2 - \frac{1}{2} n^2 \right) - \frac{1}{n} \left( 0 - 0 \right)
\]

\[
= \frac{1}{2} n^2 - \frac{1}{2} n
\]

What is the smallest positive integer \( n \) such that \( G(n) > 150 \)?

A 22
B 23
C 24
D 25
E 26

\[
G(n) = \sum_{r=1}^n r = \sum_{r=1}^n \frac{1}{2} r = \frac{1}{2} \sum_{r=1}^n r
\]

\[
= \frac{1}{2} \left( 1+2+3+\ldots+n \right)
\]

\[
= \frac{1}{2} \times \frac{1}{2} n(n+1)
\]

\[
= \frac{1}{4} n(n+1)
\]

For \( G(n) > 150 \) we need \( \frac{1}{4} n(n+1) > 150 \)

\[
n(n+1) > 600
\]

Trying some of the options, we get:

(1) For 24 we would have \( 24 \times 25 = 600 \) which isn't 7600 so doesn't work.

(2) For 25 we would have \( 25 \times 26 = 650 > 600 \).
The graph of $y = \log_{10}x$ is translated in the positive $y$-direction by 2 units. This translation is equivalent to a stretch of factor $k$ parallel to the $x$-axis.

What is the value of $k$?

- **A** 0.01
- **B** $\log_{10} 2$
- **C** 0.5
- **D** 2
- **E** $\log_2 10$
- **F** 100

So $k = 10^{-2} = \frac{1}{100} = 0.01$
The set of solutions to the inequality $x^2 + bx + c < 0$ is the interval $p < x < q$

where $b$, $c$, $p$ and $q$ are real constants with $c < 0$.

In terms of $p$, $q$ and $c$, what is the set of solutions to the inequality $x^2 + bcx + c^3 < 0$?

A \[ \frac{p}{c} < x < \frac{q}{c} \]

For $x^2 + bcx + c^3 = 0$ we would have:

\[ x = \frac{-bc \pm \sqrt{b^2c^2 - 4c^3}}{2} = \frac{-bc \pm \sqrt{c^2(b^2 - 4c)}}{2} \]

B \[ \frac{q}{c} < x < \frac{p}{c} \]

\[ x = \frac{-bc \pm c\sqrt{b^2 - 4c}}{2} \]

C \[ pc < x < qc \]

\[ x = \frac{c(-b \pm \sqrt{b^2 - 4c})}{2} \]

D \[ qc < x < pc \]

\[ x = \frac{1}{2} c (-b \pm \sqrt{b^2 - 4ac}) \] each for $x^2 + bx + c$ the roots are obviously $\frac{1}{2} b \pm \sqrt{b^2 - 4ac}$

Hence the roots of $x^2 + bcx + c^3$ are $pc$ and $qc$ so the answer must be [C] or [D]

we need to find if $pc > qc$ or $qc > pc$. We know $c < 0$ and $p < q$ so : $pc > qc$
The lengths of the sides $QR$, $RP$ and $PQ$ in triangle $PQR$ are $a$, $a + d$ and $a + 2d$ respectively, where $a$ and $d$ are positive and such that $3d > 2a$.

What is the full range, in degrees, of possible values for angle $PRQ$?

A 0 $< \text{angle } PRQ < 60$

By the cosine rule which says:

\[(a+2d)^2 = a^2 + (a+d)^2 - 2a(a+d) \cos \Theta\]

B 0 $< \text{angle } PRQ < 120$

we have

\[a^2 + 4ad + 4d^2 = a^2 + a^2 + 2ad + d^2 - 2a(a+d) \cos \Theta\]

C 60 $< \text{angle } PRQ < 120$

\[-a^2 + 2ad + 3d^2 = -2a(a+d) \cos \Theta\]

D 60 $< \text{angle } PRQ < 180$

\[\cos \Theta = \frac{-a^2 + 2ad + 3d^2}{-2a(a+d)}\]

\[= \frac{a^2 - 2ad - 3d^2}{2a(a+d)}\]

\[= \frac{(a+d)(a-3d)}{2a(a+d)}\]

\[= \frac{a-3d}{2a}\]

Since $3d > 2a$, $-a > a-3d$ so $\cos \Theta < \frac{-a}{2a} = \frac{-1}{2}$

so $\Theta > 120^\circ$

END OF TEST
We are Cambridge Assessment Admissions Testing, part of the University of Cambridge. Our research-based tests provide a fair measure of skills and aptitude to help you make informed decisions. As a trusted partner, we work closely with universities, governments and employers to enhance their selection processes.

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