

STEP 2019 Paper 3 Question 5

The 2019 STEP 3 question paper contained a typographical error. This document describes the measures taken to mitigate and quantify the impact of the error.

The question says:

5 (i) Let

$$f(x) = \frac{x}{\sqrt{x^2 + p}},$$

where p is a non-zero constant. Sketch the curve $y = f(x)$ for $x \geq 0$ in the case $p > 0$.

(ii) Let

$$I = \int \frac{1}{(b^2 - y^2)\sqrt{c^2 - y^2}} dy,$$

where b and c are positive constants. Use the substitution $y = \frac{bx}{\sqrt{x^2 + p}}$, where p is a suitably chosen constant, to show that

$$I = \int \frac{1}{b^2 + (b^2 - c^2)x^2} dx.$$

Evaluate

$$\int_1^{\sqrt{2}} \frac{1}{(3 - y^2)\sqrt{2 - y^2}} dy.$$

[Note: $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + \text{constant}$.]

Hence evaluate

$$\int_{\frac{1}{\sqrt{2}}}^1 \frac{y}{(3y^2 - 1)\sqrt{2y^2 - 1}} dy.$$

(iii) By means of a suitable substitution, evaluate

$$\int_{\frac{1}{\sqrt{2}}}^1 \frac{1}{(3y^2 - 1)\sqrt{2y^2 - 1}} dy.$$

The typo occurs at the start of part (ii). The substitution should read $y = \frac{cx}{\sqrt{x^2 + p}}$

To analyse the possible impact of the typo, we can divide the question into sections:

A	Part (i)
B	Part (ii) showing substitution gives given I
C	Part (ii) The "Evaluate" integral with limits 1 and $\sqrt{2}$
D	Part (ii) Hence evaluate ...
E	Part (iii)

Part A: unaffected by the typo

Part B: inaccessible at it stands as the typo prevents candidates from deriving the given expression. [Many derived correctly the result of the *given* substitution.]

Part C: is accessible without the previous part - primarily by trusting the previous result from the given substitution [or equivalently by devising a suitable substitution]; this would lead to the correct answer, but might not use the arctan integral given as a hint.

If a candidate were to attempt to evaluate the integral by using the given result and setting $b = \sqrt{3}$ and $c = \sqrt{2}$ [as would be expected] they would be hampered by the fact there is not enough information to deal with limits numerically.

Part D: This section is accessible independently of the typo, but it is clear that the *intention* of the question was that this would follow from the previous parts once an appropriate intermediate sub had been identified.

Part E: This is accessible independently of the typo

Devising a mark scheme

We devised a mark scheme that would reward good mathematics regardless of whether candidates had spotted and corrected the typo or not. We based our mark scheme on a number of principles:

1. Candidates who **did** spot the typo should not be disadvantaged by any alternative mark scheme.
2. Candidates who **did not** spot the typo should be maximally rewarded for demonstrating good mathematical understanding and appropriate mathematical skills that relate to the assessment aims of the question.
3. A candidate who did not spot the typo should still be able to score full-marks.
4. Be aware that, whilst much of part (ii) is strictly accessible independently of whether the typo was noticed or not, it would not be fair to penalise students who did not realise this.
5. Finally, on *assuming* much of part (ii) is inaccessible directly following the typo, we should award only method marks for this section in the alternative mark scheme.

Marking question 5

The question was marked by a small team of highly experienced markers – hand-picked for the task. The marking team were supervised by the chief marking supervisor [a very experienced examiner, who also wrote the mark scheme] together with a mathematics assessment expert.

For question 5, we suspended the rule that crossed out work is not to be marked. All crossed out work and rough work for question 5 was assessed and each candidate was awarded marks based on their best attempts.

Qualitative flagging

After the marking process was completed all scripts that included any attempt at question 5 were individually examined by either a senior examiner or a senior mathematics assessment expert. Every script was flagged using the following:

- 0 No evidence that the candidate was adversely affected
- 1 Some evidence that the candidate was adversely affected, but that this was most likely minor – maybe a little lost time
- 2 Evidence that the candidate was adversely affected in some way – e.g. by attempting the question multiple times etc. – and some more significant loss of time might have resulted

In flagging cases, we have erred on the side of generosity – so that, if there was doubt between giving a flag of 0 or 1, or 1 or 2, we always gave the higher flag.

In addition, almost all flagged cases are accompanied by a brief comment.

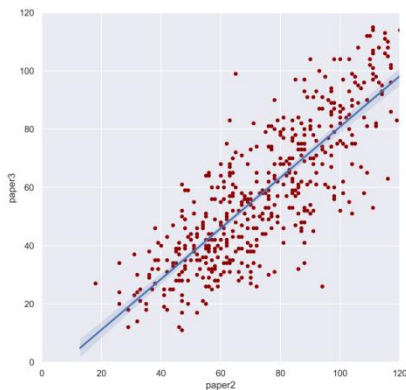
This information was provided to the official STEP users, the Universities of Cambridge and Warwick, so that it could be considered when reviewing applicants' results.

Quantitative analysis

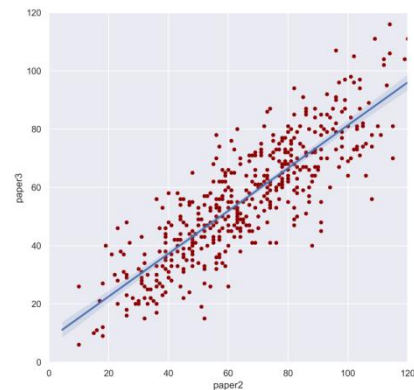
In order to determine the effects of the typo on candidate performance at population level, we carried out a series of analyses.

Firstly, we plotted the regression relationship between performance on STEP 2 and STEP 3 for 2019 and compared it to the relationship for 2018 as a baseline, also calculating the Pearson correlation between scores:

2018 (Pearson = 0.816)



2019 (Pearson = 0.844)



These plots, and in particular the stronger correlation between STEP 2 and STEP 3, do not indicate any population-level effect – the non-construct related nature of any knock-on effects from Q5 (in particular lack of time and affective issues such as loss of confidence) should result in a weaker correlation.

Secondly, in order to quantify any effects on performance on the rest of the test, we considered candidates for whom Q5 represented a top-six score and calculated their total score on the remaining five items. We then compared this score with those for candidates for whom Q5 did not represent a top-six score with one question removed at random. This did not indicate any disadvantage across the population of candidates who attempted Q5; on repeating this analysis for all other items, there was again no indication that Q5 resulted in lower performance.

A lack of population-level effects does not, however, mean that individual candidates were not disadvantaged. In order to supplement the qualitative analysis described above, we calculated the residuals from the regression line using STEP 2 as a predictor to identify candidates whose STEP 3 performance was notably below that predicted by STEP 2 based on the residual (actual STEP 3 score minus predicted STEP 3 score); this information was included alongside the 0–2 flagging system as a further indicator that a candidate may have been affected by the typo.

Cambridge Assessment Admissions Testing

25th July 2019